DMV-PTM Mathematical Meeting 17–20.09.2014, Poznań Galois representations attached to étale coho-

Sebastian Petersen

mology

Universität der Bundeswehr, Neubiberg, Germany sebastian.petersen@unibw.de

Session: 3. Arithmetic Geometry

Let K be a field and X/K a separated algebraic scheme. Grothendieck and Artin constructed for every prime number ℓ and every $i \in \mathbf{N}$ a cohomology group $H^i(X_{\overline{K}}, \mathbf{Q}_{\ell})$ which comes with a natural action of the Galois group $\operatorname{Gal}(\overline{K}/K)$; one thus obtains a Galois representation

$$\rho_{\ell} \colon \operatorname{Gal}(\overline{K}/K) \to \operatorname{Aut}_{\mathbf{Q}_{\ell}}(H^{i}(X_{\overline{K}}, \mathbf{Q}_{\ell})).$$

Denote by $\rho: \operatorname{Gal}(\overline{K}/K) \to \prod_{\ell} \operatorname{im}(\rho_{\ell})$ the homomorphism induced by the ρ_{ℓ} . Serre proved recently that in the case where K is a number field the family $(\rho_{\ell})_{\ell}$ is almost independent in the following sense: There exists a finite Galois extension K'/K such that

$$\rho(\operatorname{Gal}(\overline{K}/K')) = \prod_{\ell} \rho_{\ell}(\operatorname{Gal}(\overline{K}/K')).$$

This information is quite useful when working with such families of ℓ -adic representations attached to schemes, and it ties in well with the adelic openness conjecture. There are analogous results in the case where K is an arbitrary finitely generated field of characteristic zero, and where K is a geometric function field of arbitrary characteristic. Following a suggestion of Illusie and making strong use of results of Orgogozo this was used to establish a quite general independence theorem for families $(F_{\ell})_{\ell}$ of étale sheaves of \mathbf{F}_{ℓ} -vector spaces over an arithmetic scheme which satisfy a uniform constructability and a potential semistability condition.