ESSENTIAL COHOMOLOGY CLASSES AND SECTIONAL CATEGORY OF SUBGROUP INCLUSIONS

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The Lusternik-Schnirelmann category of a space and Farber's topological complexity [3] are particular examples of a more general notion, the *sectional category*, introduced by Schwarz in [6]. A famous theorem due to Eilenberg and Ganea, [2] gives a characterization of the LS category of an aspherical space as the cohomological dimension of its fundamental group (for dimension greater than 3). In the context of topological complexity, the generalization of the theorem of Eilenberg and Ganea, or any other algebraic characterization of the TC of aspherical spaces, remain as one of the most interesting open problems.

In [4] the authors developed the notion of *essential cohomology classes* and used it to investigate bounds for topological complexity. The key part of their work is the construction of a spectral sequence, which provides a sequence of obstructions for a cohomology class to be essential. In [1], the notion of sectional category of group monomorphisms was introduced and studied. The sectional category of a subgroup inclusion $H \hookrightarrow G$ can be defined as the sectional category of the corresponding map between Eilenberg-MacLane spaces. In particular, the usual topological complexity of a group can be seen as $\mathrm{TC}(G) = \mathrm{secat}(\Delta G \hookrightarrow G \times G)$.

In this ongoing project, we generalize the notion of essential cohomology classes to arbitrary monomorphisms of groups, including the construction of a spectral sequence which encodes the obstruction groups for a class to be essential. Then we use them to study the sectional category of subgroup inclusions, and to provide new lower bounds. Finally, we discuss how these new bounds specialize to the case of usual topological complexity of groups, and the topological complexity of group epimorphisms, as defined by [5].

References

[1] Z. Błaszczyk, J. Carrasquel-Vera, and A. Espinosa Baro: *On the sectional category of subgroup inclusions and Adamson cohomology theory*, Journal of Pure and Applied Algebra 226.6, p. 106959, 2022.

[2] S. Eilenberg and T. Ganea: *On the Lusternik-Schnirelmann category of abstract groups*, Ann. of Math. (2), 65: 517–518 1957.

[3] M. Farber: *Topological complexity of motion planning*, Discrete Comput. Geom., 29(2):211–221, 2003.

[4] M. Farber and S. Mescher: *On the topological complexity of aspherical spaces*, J. Topol. Anal. 12.2, pp. 293–319, 2020.

[5] M. Grant: *Parametrised topological complexity of group epimorphisms*, Topol. Methods Nonlinear Anal. 60.1, pp. 1049–1075, 2022.

[6] A. Schwarz: *The genus of a fiber space*, A.M.S Transl., 55:49–140, 1966.