An analogue of Weil's program for the Riemann zeta function and Dirichlet L-functions

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Joint work with Yoichi Uetake.

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In the 1940s A. Weil used the intersection theory on surfaces to prove the Riemann hypothesis for curves over finite fields. In our work we introduce axioms of (what we call) abstract intersection theory for an operator $A\colon H \to H$ in the Hilbert space H. These axioms are analogous to the properties of the Frobenius morphism used in Weil's theory. In our work we investigate a special class of operators A with spectrum, which is only the point spectrum, located in the critical strip $0 < Res < 1$. We say that A satisfies the Riemann hypothesis if its spectrum is on the critical line $Res = 1/2$. We constructed two special models of the abstract intersection theory: the GNS (Gelfand-Naimark-Segal) model and the standard model. We proved that the axioms of the standard model for A are satisfied if and only if the Riemann hypothesis for A is true and the semi-simplicity property for A holds. Similar results can be proven for the GNS model. These results can be applied to the investigation of nontrivial zeros of the Riemann zeta function and the Dirichlet L-functions. Namely for every Dirichlet character, using the method of automorphic scattering, Yoichi Uetake constructed an operator A with spectrum equal to the set of nontrivial zeros (counting with multiplicities) of the corresponding Dirichlet L-function. In particular, for the trivial character, this construction concerns the Riemann zeta function. As a consequence we can show that a Dirichlet L-function (including the Riemann zeta-function) satisfies the Riemann hypothesis and its all nontrivial zeros are simple if and only if the axioms of the corresponding standard or GNS model are satisfied.