Compatible systems of Galois representations in positive characteristic.

(Joint work with Gebhard Böckle and Wojciech Gajda)

Let $K/\mathbb{F}_p$ be a finitely generated extension. A *compatible system over* K is a family of representations $(\rho_\ell : \mathrm{Gal}(K) \to \mathrm{GL}_{V_\ell}(\mathbb{Q}_\ell))_{\ell \neq p}$ indexed by prime numbers $\neq p$ and satisfying a certain compatibility condition at the the Frobenius elements of the places of K. (The details will be explained it the talk.) The most important source for compatible systems is étale cohomology: If X/K is a smooth proper variety and $V_\ell = H^j(X_{\overline{K}}, \mathbb{Q}_\ell)$, then the family of representations $(\mathrm{Gal}(K) \to \mathrm{GL}_{V_\ell}(\mathbb{Q}_\ell))_{\ell \neq p}$ describing the action of $\mathrm{Gal}(K)$ on $H^j(X_{\overline{K}}, \mathbb{Q}_\ell)$ is a compatible system. In this talk we will mainly focus on the aspect that groundbreaking work of L. Lafforgue paved the way for substantial progress concerning such compatible systems of Galois representations in positive characteristic.