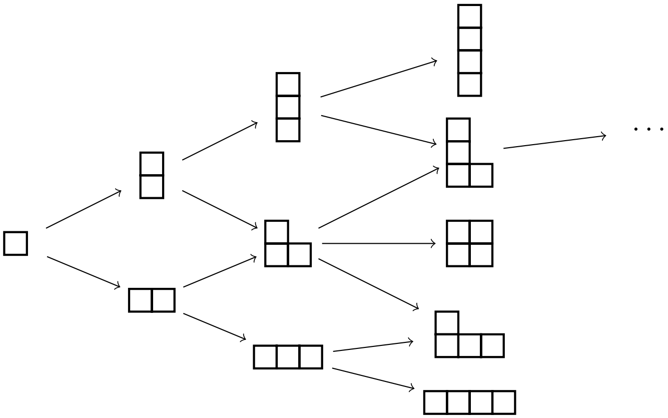
Jeu de taquin

travail commun avec Dan Romik

Piotr Śniady

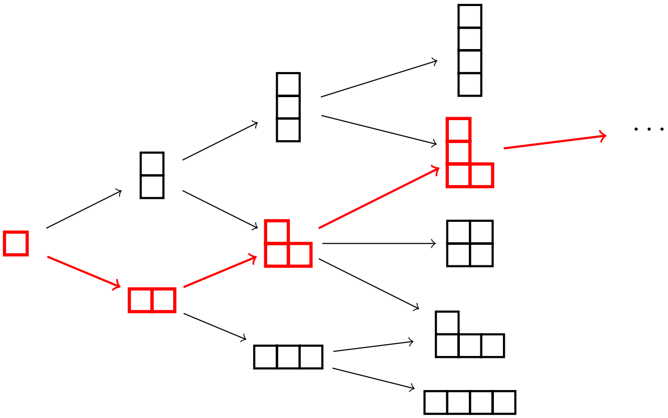
Université de Poznań
et
Académie Polonaise des Sciences

Young graph: irreducible representations of symmetric groups $S_1 \subset S_2 \subset S_3 \subset \dots$



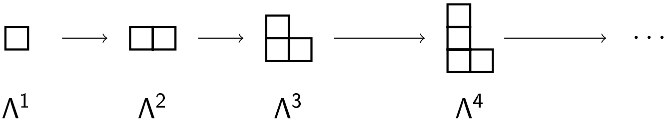
Tool for studying S_∞

Young graph: irreducible representations of symmetric groups $S_1 \subset S_2 \subset S_3 \subset \dots$

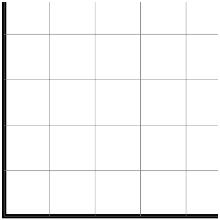


Tool for studying S_∞

paths in Young graph $\longleftrightarrow$ tableaux



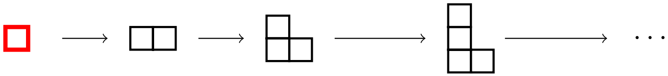
infinite path in Young graph



infinite tableau

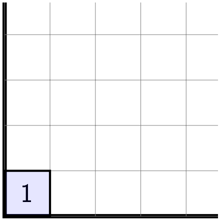
$\Omega :=$ set of infinite tableaux / set of infinite paths

paths in Young graph $\longleftrightarrow$ tableaux



Λ^1 Λ^2 Λ^3 Λ^4

infinite path in Young graph



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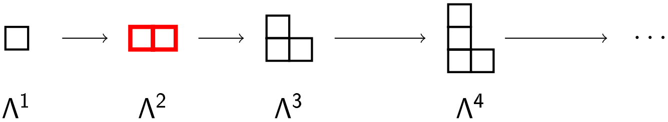

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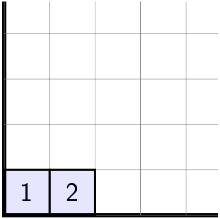

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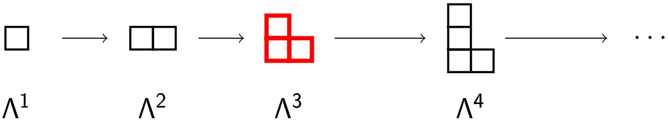
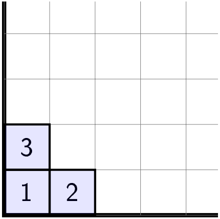


infinite path in Young graph



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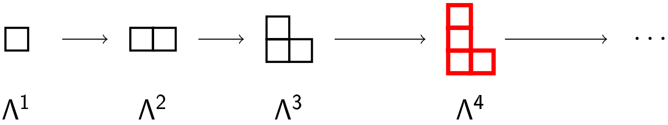

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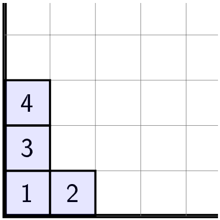

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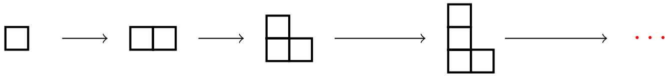


Λ^1 $\quad$ Λ^2 $\quad$ Λ^3 $\quad$ Λ^4

infinite path in Young graph

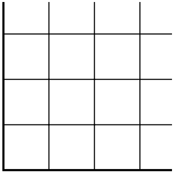
⋮		⋮		
6	15	21	24	
4	12	17	19	⋯
3	5	8	11	
1	2	7	9	⋯

infinite tableau

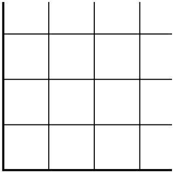
$\Omega :=$ set of infinite tableaux / set of infinite paths

infinite Robinson-Schensted-Knuth (RSK) map

$$\text{infinite word} \overset{\text{RSK}}{\longmapsto} \text{recording tableau}$$



insertion tableau

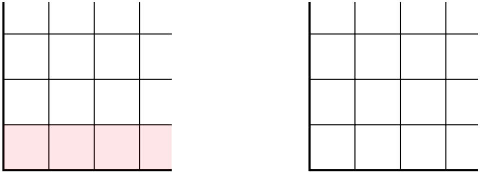


recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau



insertion tableau recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

① *start from the first row,*

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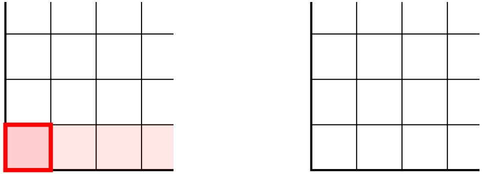

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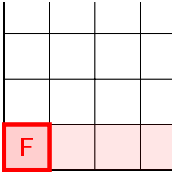

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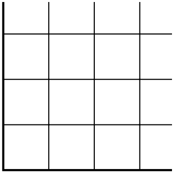

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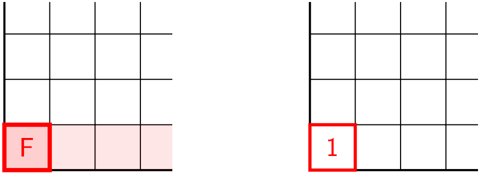

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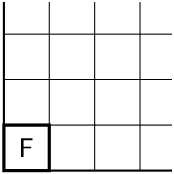

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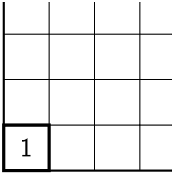

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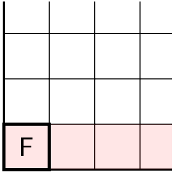

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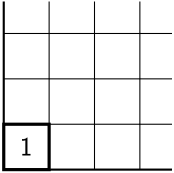

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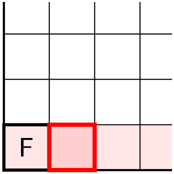

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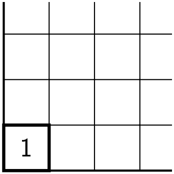

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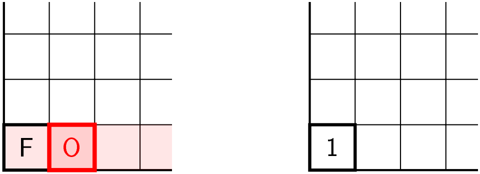

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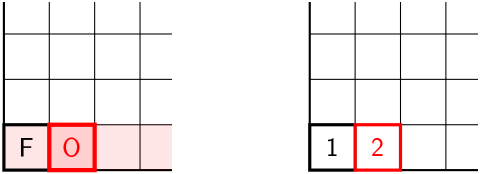

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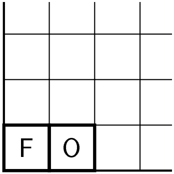

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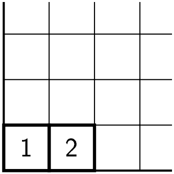

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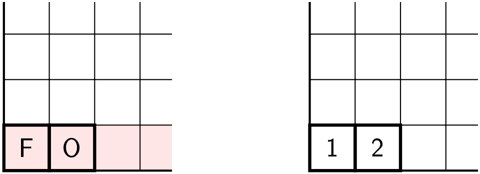

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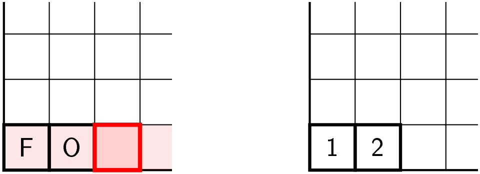

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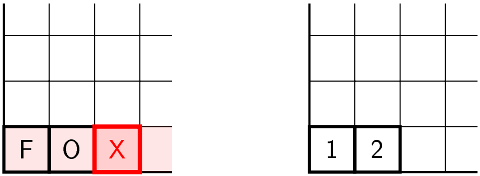

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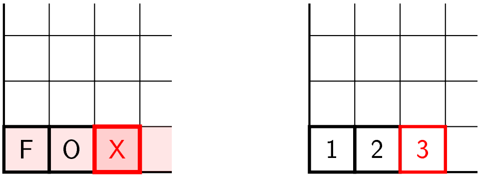

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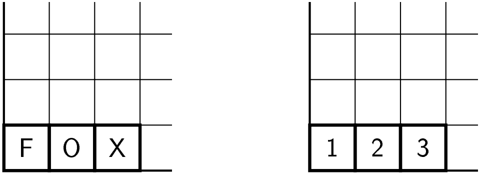

insertion tableau | recording tableau

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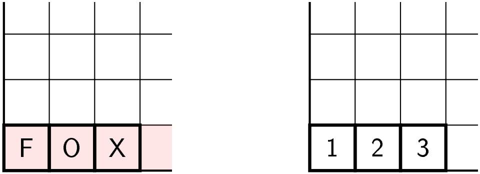

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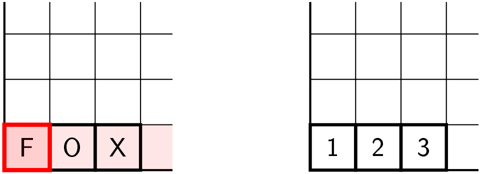

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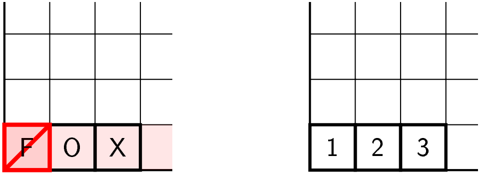

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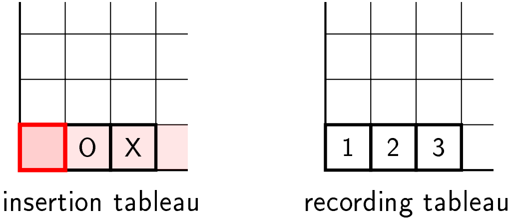

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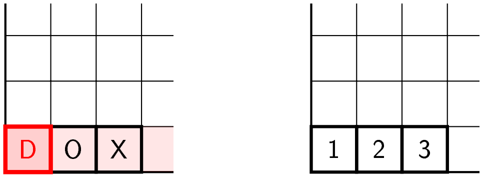

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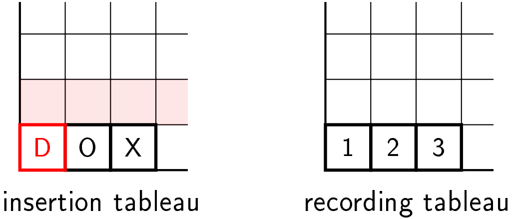

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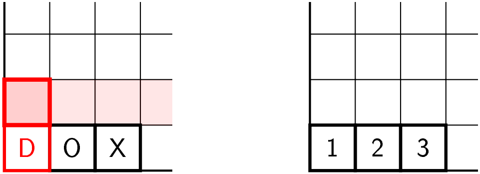

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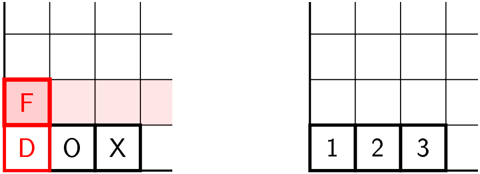

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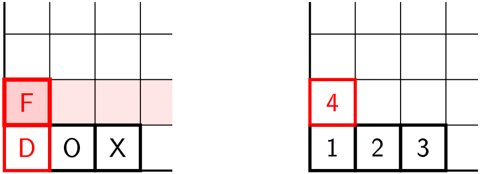

insertion tableau   recording tableau

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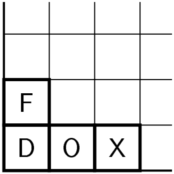
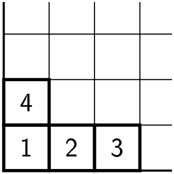
infinite Robinson-Schensted-Knuth (RSK) map

infinite word $\overset{\text{RSK}}{\mapsto}$ recording tableau



insertion tableau



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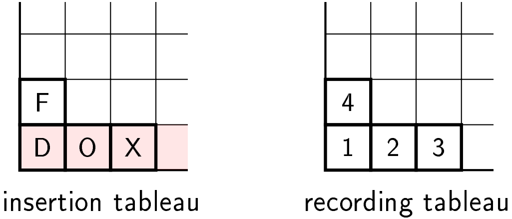

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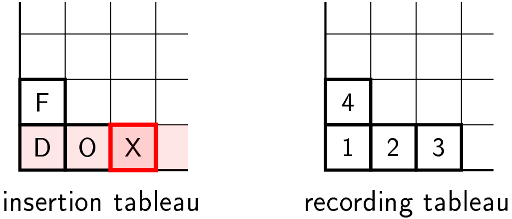

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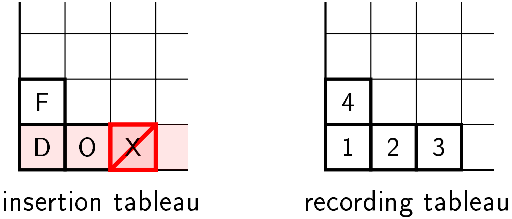

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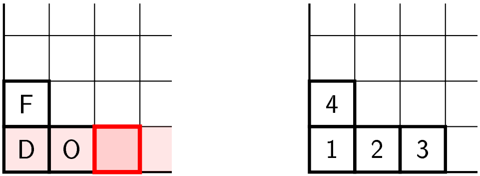

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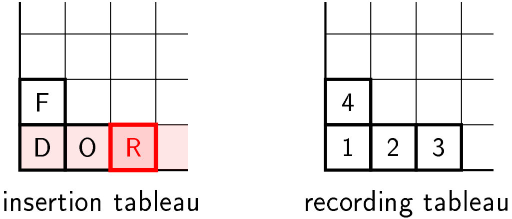

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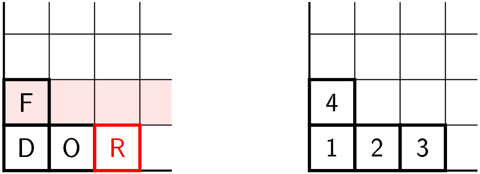

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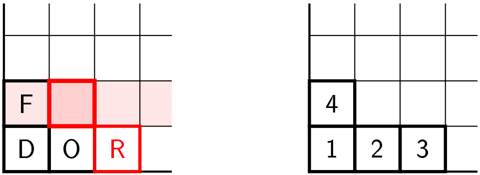

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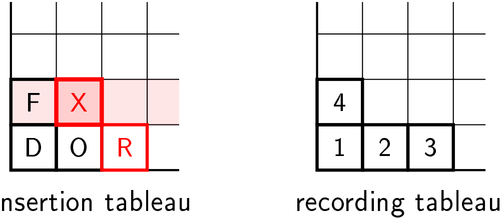

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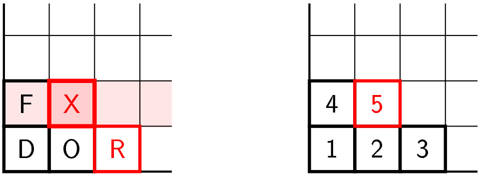

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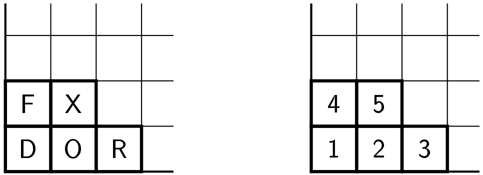

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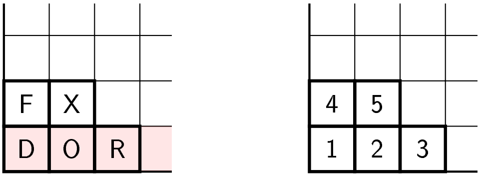

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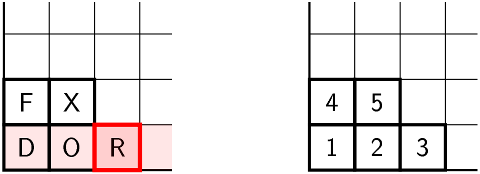

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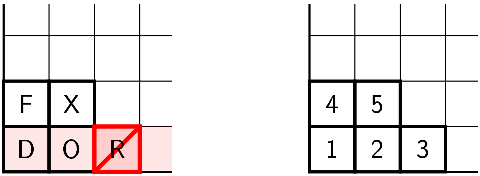

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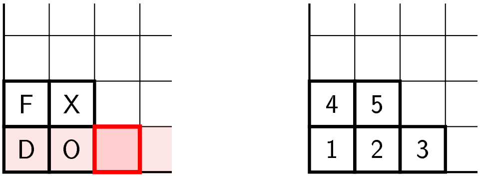

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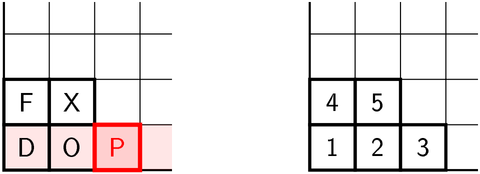

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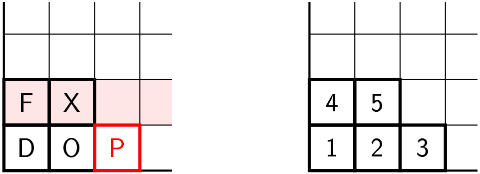

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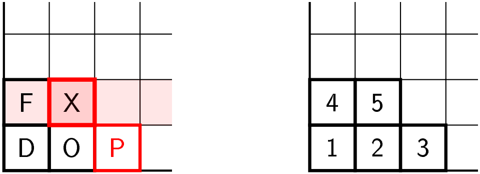

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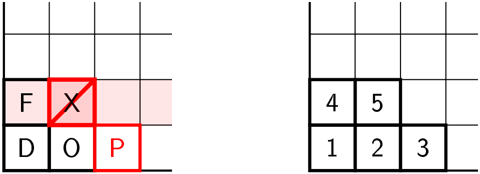

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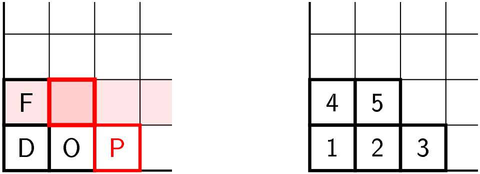

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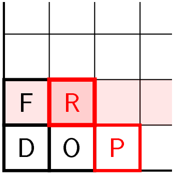

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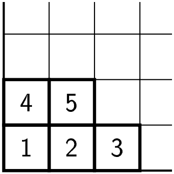

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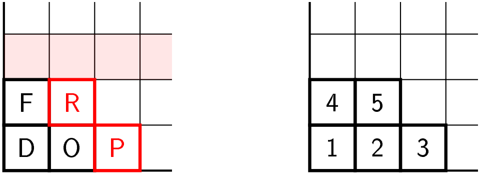

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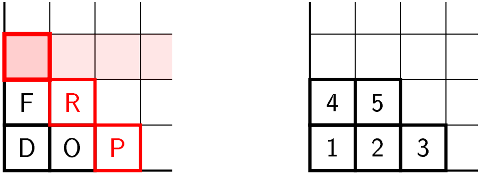

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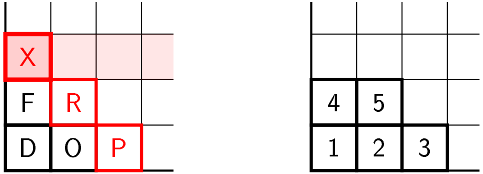

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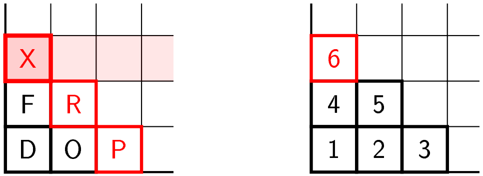

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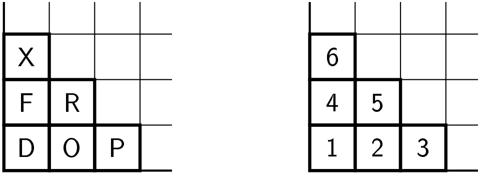

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F	L	N	T
D	G	M	S
B	E	J	Q
A	C	H	I

insertion tableau

7	16	22	25
6	10	14	24
4	5	9	17
1	2	3	8

recording tableau

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$$\text{infinite word} \stackrel{\text{RSK}}{\longmapsto} \text{recording tableau}$$

7	16	22	25
6	10	14	24
4	5	9	17
1	2	3	8

~~insertion tableau~~ recording tableau

F O X D R P B Z U L G E A T W N S M Y V C J H Q I K

if $X_0, X_1, \dots$ are independent $U(0,1)$ random variables then

$$\text{RSK}(X_0, X_1, \dots) \stackrel{\text{distribution}}{=} \text{Plancherel measure}$$

infinite Robinson-Schensted-Knuth (RSK) map

$$(x_0, x_1, \dots) \overset{\text{RSK}}{\longmapsto} t$$

infinite Robinson-Schensted-Knuth (RSK) map

$$(x_0, x_1, \ldots)$$

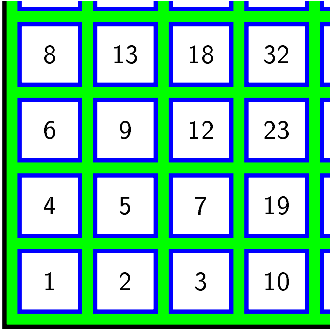
$$\text{RSK} \Big\downarrow \;\; \Big\uparrow \text{inverse?}$$

$$t$$

8	13	18	32
6	9	12	23
4	5	7	19
1	2	3	10

jeu de taquin

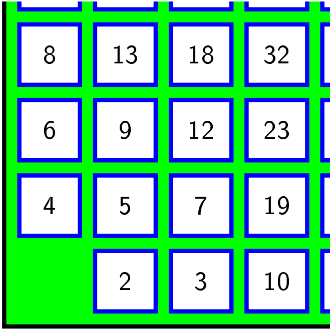
① start with $t \in \Omega$,



jeu de taquin

① start with $t \in \Omega$,

② remove corner box,



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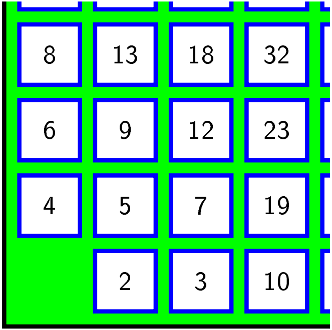

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① start with $t \in \Omega$,

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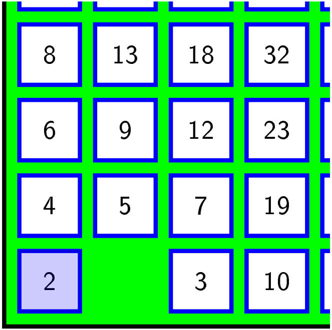
③ sliding,



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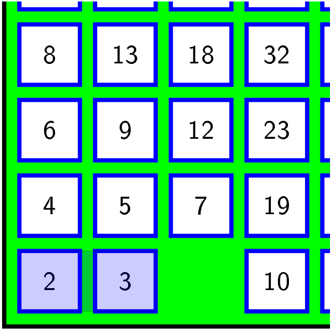

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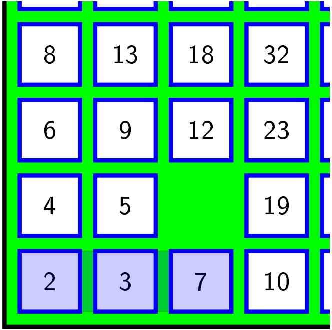

jeu de taquin

① start with $t \in \Omega$,

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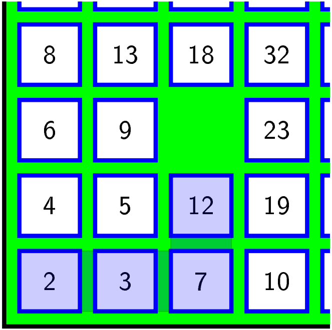

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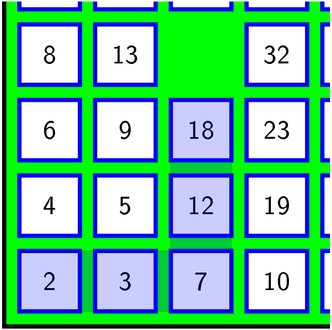

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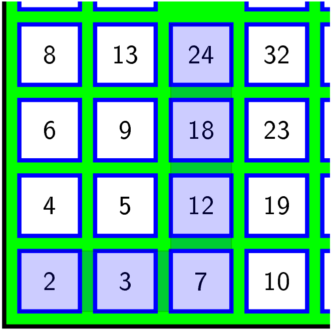

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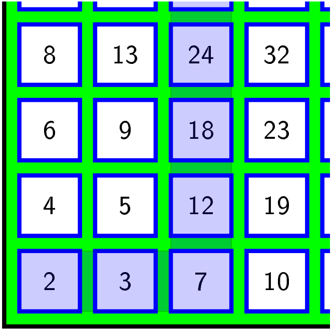

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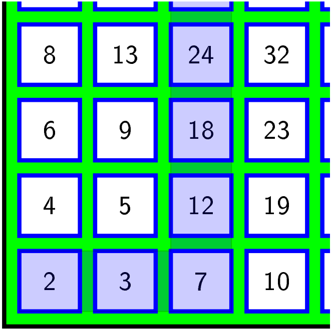
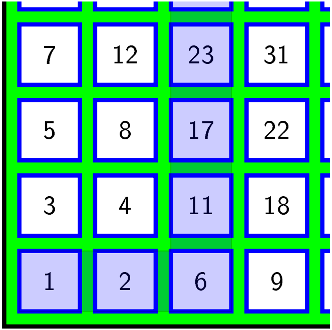


jeu de taquin

① start with $t \in \Omega$,
② remove corner box,
③ sliding,
④ subtract 1 from all boxes



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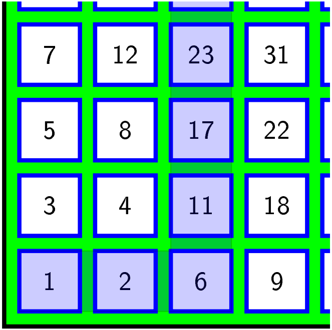

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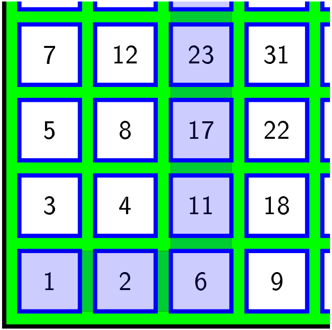
output:

- new tableau $J(t)$,
- blue trajectory $\mathbf{c}(t) = (c_1, c_2, \dots)$



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'how representation of $S_{\{1,2,3,\ldots\}}$ is related to its restriction to $S_{\{2,3,\ldots\}}$?'

jeu de taquin - overview

8	13	18	32
6	9	12	23
4	5	7	19
1	2	3	10

original tableau t

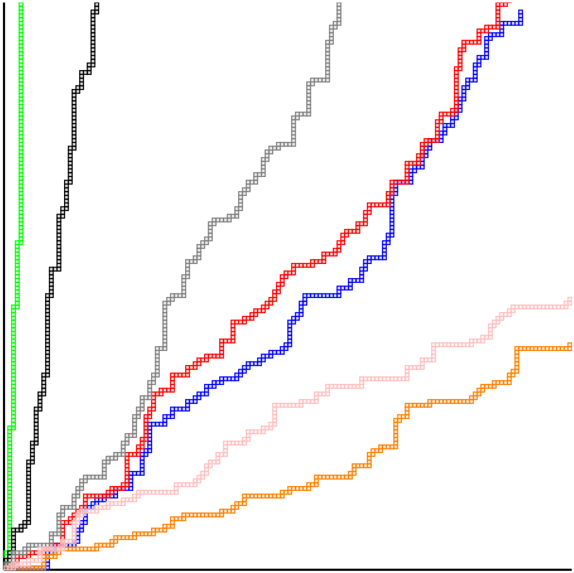
8	13	24	32
6	9	18	23
4	5	12	19
2	3	7	10

outcome of slidings

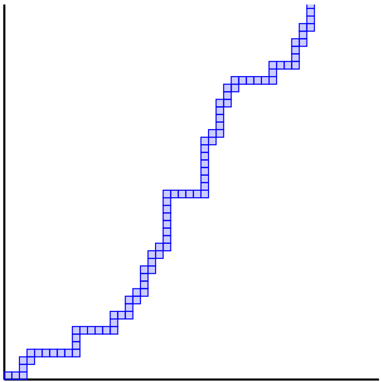
7	12	23	31
5	8	17	22
3	4	11	18
1	2	6	9

new tableau $J(t)$

trajectories of jeu de taquin



trajectories of jeu de taquin



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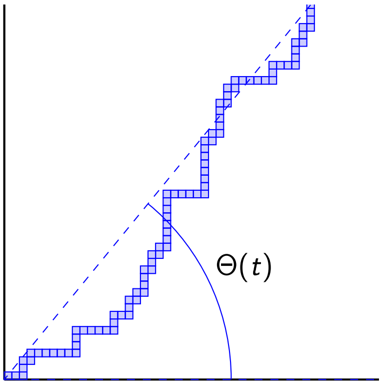
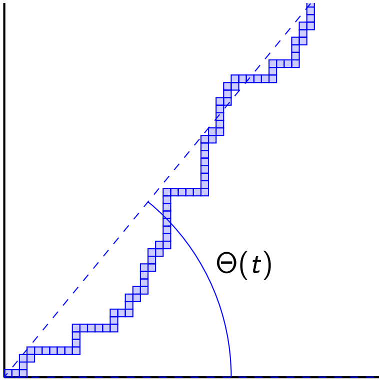


trajectories of jeu de taquin



if $t = \mathrm{RSK}(X_0, X_1, \dots) \in \Omega$ is random, Plancherel distributed

then its jdt trajectory $\mathbf{c}(t)$ is almost surely asymptotically a straight line,

i.e.

$$\lim_{k\to\infty} \frac{c_k}{\|c_k\|} = \big(\cos\Theta(t), \sin\Theta(t)\big)$$

exists almost surely

$$\begin{array}{ccc} t_0 & \overset{J}{\longmapsto} & t_1 \\ \Theta \big\downarrow & & \\ \theta_0 & & \end{array}$$

$$\begin{array}{ccccc} t_0 & \overset{J}{\longmapsto} & t_1 & \overset{J}{\longmapsto} & \cdots \\ \Theta \downarrow & & \Theta \downarrow & & \\ \theta_0 & & \theta_1 & & \end{array}$$

$$
\begin{array}{ccccc}
t_0 & \overset{J}{\longmapsto} & t_1 & \overset{J}{\longmapsto} & \cdots \\
\Theta \big\downarrow & & \Theta \big\downarrow & & \\
\theta_0 & & \theta_1 & &
\end{array}
$$

jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$

$$
\begin{array}{ccccc}
(x_0, x_1, \ldots) & & & & \\
\big\downarrow \scriptstyle{\mathrm{RSK}} & & & & \\
t_0 & \overset{J}{\longmapsto} & t_1 & \overset{J}{\longmapsto} & \cdots \\
\big\downarrow \scriptstyle{\Theta} & & \big\downarrow \scriptstyle{\Theta} & & \\
\theta_0 & & \theta_1 & &
\end{array}
$$

jeu de taquin dynamical system $(\Omega, \text{Plancherel}, J)$

$$\begin{array}{ccccc}
(x_0, x_1, \dots) & \overset{?}{\longmapsto} & ? & \overset{?}{\longmapsto} & \cdots \\
\text{RSK} \downarrow & & \text{RSK} \downarrow & & \\
t_0 & \overset{J}{\longmapsto} & t_1 & \overset{J}{\longmapsto} & \cdots \\
\Theta \downarrow & & \Theta \downarrow & & \\
\theta_0 & & \theta_1 & &
\end{array}$$

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\end{array}
$$

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i.i.d. shift dynamical system $\left([0,1]^{\mathbb{N}}, \prod \text{Lebesgue}, s\right)$

$$
\begin{array}{ccccc}
(x_0, x_1, \dots) & \overset{s}{\longmapsto} & (x_1, x_2, \dots) & \overset{s}{\longmapsto} & \cdots \\
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t_0 & \overset{J}{\longmapsto} & t_1 & \overset{J}{\longmapsto} & \cdots \\
\downarrow \scriptstyle\Theta & & \downarrow \scriptstyle\Theta & & \\
\theta_0 & & \theta_1 & &
\end{array}
$$

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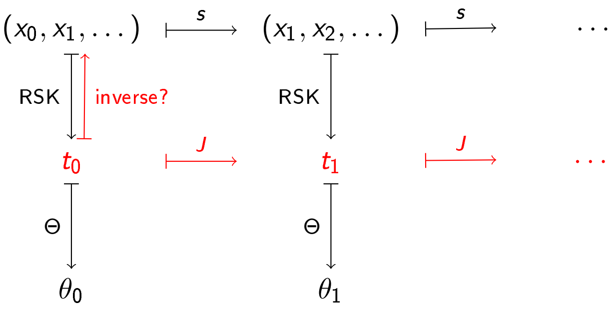

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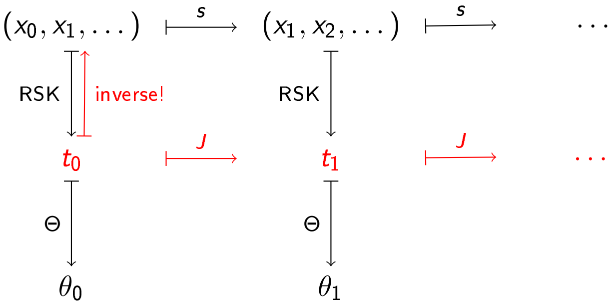

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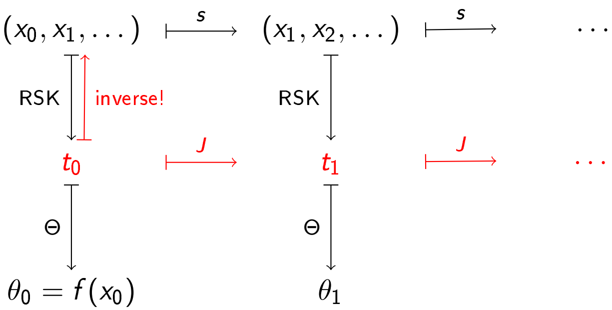

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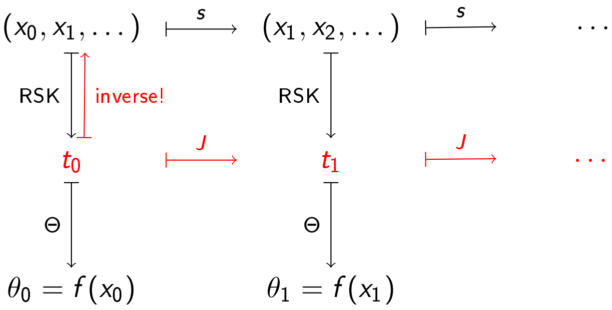

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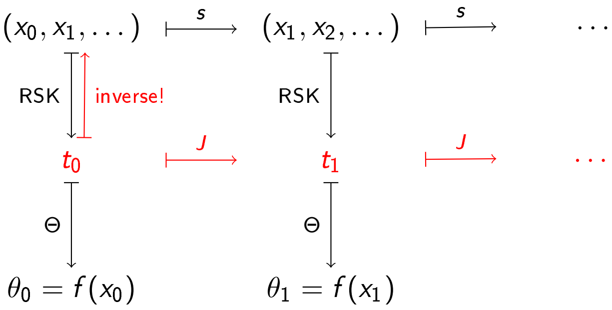


jeu de taquin dynamical system $\left(\Omega, \text{Plancherel}, J\right)$

the jeu de taquin dynamical system is isomorphic to i.i.d. shift

the inverse map is given by $x_i = f^{-1}(\theta_i)$

some consequences of the isomorphism:

- jdt is a measure-preserving transformation,
- jdt is ergodic,
- slope angles $\theta_0, \theta_1, \ldots$ are independent random variables (put paths $\mathbf{c}(t_0), \mathbf{c}(t_1), \ldots$ are not independent),
- generalizations to other probability measures on Ω / other representations of S_∞,

why Θ exists and is a function of x_0?

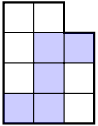
x_0 is fixed

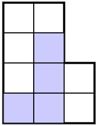
$x_1, x_2, \dots$ are random, independent $U(0,1)$

$$(x_0, x_1, \dots) \overset{s}{\longmapsto} (x_1, x_2, \dots)$$

$$\Big\downarrow \text{RSK} \qquad\qquad\qquad\qquad \Big\downarrow \text{RSK}$$

$$\Lambda^1 \nearrow \Lambda^2 \nearrow \dots \overset{J}{\longmapsto} \overline{\Lambda}^1 \nearrow \overline{\Lambda}^2 \nearrow \dots$$

$\Lambda^{n+1} =$ 

$\overline{\Lambda}^n =$ 

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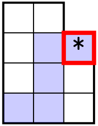
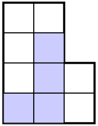
\downarrow \text{RSK} & & \downarrow \text{RSK} \\
\Lambda^1 \nearrow \Lambda^2 \nearrow \dots & \stackrel{J}{\longmapsto} & \overline{\Lambda}^1 \nearrow \overline{\Lambda}^2 \nearrow \dots
\end{array}$$

$\Lambda^{n+1} =$ 

$\overline{\Lambda}^n =$ 

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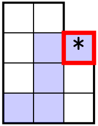
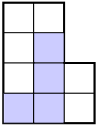
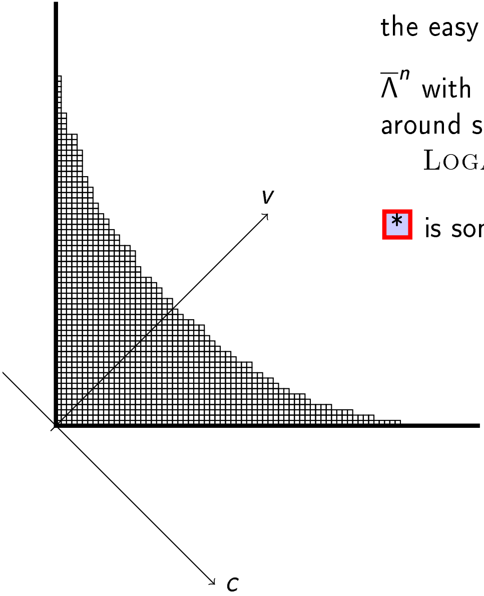
$\Lambda^{n+1} =$ 

$\overline{\Lambda}^n =$ 

$$\boxed{*} = \Lambda^{n+1} \setminus \overline{\Lambda}^n = \text{RSK}(x_0, x_1, \dots, x_n) \setminus \text{RSK}(x_1, \dots, x_n)$$

Is it true that asymptotically position of $\boxed{*}$ depends only on x_0?



the easy part:

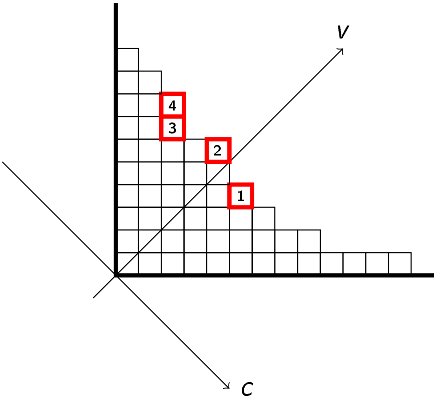
$\overline{\Lambda}^n$ with high probability concentrates around some limit shape

LOGAN, SHEPP, VERSHIK, KEROV

$\boxed{*}$ is somewhere on the boundary of $\overline{\Lambda}^n$

instead of (for deterministic x_0) studying

$$\mathrm{RSK}(x_0, x_1, \ldots, x_n) \setminus \mathrm{RSK}(x_1, \ldots, x_n) = \boxed{*}$$



we study (for random $0 < t_1 < \cdots < t_k < 1$)

$$\mathrm{RSK}(t_1, \ldots, t_k, x_1, \ldots, x_n) \setminus \mathrm{RSK}(x_1, \ldots, x_n) = \{\boxed{1}, \ldots, \boxed{k}\}$$

plactic Littlewood-Richarson rule

if $0 \leq x_1, \dots, x_n \leq 1$ is a random sequence, conditioned in such a way that

$$\text{shape of } \text{RSK}(x_1, \dots, x_n) = \lambda;$$

and $0 \leq t_1, \dots, t_k \leq 1$ is a random sequence, conditioned in such a way that

$$\text{shape of } \text{RSK}(t_1, \dots, t_k) = \mu;$$

then the random Young diagram

$$\text{shape of } \text{RSK}(t_1, \dots, t_k, x_1, \dots, x_n)$$

has the same distribution as random irreducible component of

$$V^\lambda \otimes V^\mu \uparrow_{S_n \times S_k}^{S_{n+k}}$$

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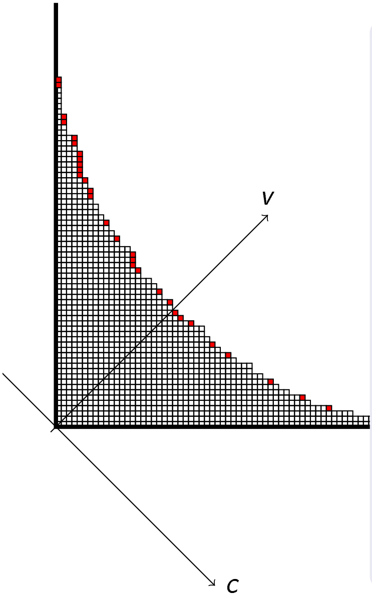
$$\text{shape of } \operatorname{RSK}(t_1, \dots, t_k, x_1, \dots, x_n)$$

has the same distribution as random irreducible component of

$$V^{\lambda} \otimes V^{(k)} \uparrow^{S_{n+k}}_{S_n \times S_k}$$

growth of Young diagrams and Jucys-Murphy elements



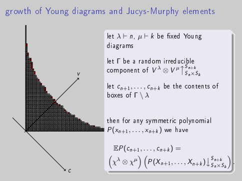
let $\lambda \vdash n$, $\mu \vdash k$ be fixed Young diagrams

let Γ be a random irreducible component of $V^\lambda \otimes V^\mu \big\uparrow_{S_n \times S_k}^{S_{n+k}}$

let $c_{n+1}, \dots, c_{n+k}$ be the contents of boxes of $\Gamma \setminus \lambda$

then for any symmetric polynomial $P(x_{n+1}, \dots, x_{n+k})$ we have

$$\mathbb{E} P(c_{n+1}, \dots, c_{n+k}) = \left(\chi^\lambda \otimes \chi^\mu \right) \left(P(X_{n+1}, \dots, X_{n+k}) \big\downarrow_{S_n \times S_k}^{S_{n+k}} \right).$$



$X_1, X_2, \dots, X_{n+k}$ are Jucys-Murphy elements;
$X_i = (1,i) + (2,i) + \cdots + (i-1,i) \in \mathbb{C}[S_{n+k}]$; they all commute; it was shown by Okounkov and Vershik that they are the key tool for understanding of the representation theory of the symmetric groups.

Dan Romik, Piotr Śniady
Jeu de taquin dynamics on infinite Young tableaux and second class particles
Ann. Probab, Volume 43, Number 2 (2015), 682-737

Piotr Śniady.
Robinson-Schensted-Knuth algorithm, jeu de taquin and Kerov-Vershik measures on infinite tableaux.
SIAM J. Discrete Math. 28 (2014), no. 2, 598-630.